Nominal and Real Interest Rates
The Fisher Equation

Author: Jacopo Tozzo
Bocconi University – PhD in Economics and Finance

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The Rate of Return

Whenever the discussion includes interest rates, assets, inflation, ecc. we are facing intertemporal economic problems.
Agents face intertemporal problems whenever they need to transfer purchasing power into the future. The way agents transfer purchasing power is by acquiring wealth (saving) that can be held in the form of bonds, stocks, cash, real estate and so on.
Since the object of saving is transferring purchasing power into the future, one of the main indicator measuring the degree asset's desirability, is its **rate of return**, i.e. the percentage increase in its value over time.
Most of assets do not guarantee with certainty their future value, therefore an expected future value of that asset is considered. In this case, the return of the asset is called **expected return**.
Measuring an asset's rate of return means comparing the changing value of the asset between two dates. But how the value of each period is measured? When the value is expressed in term of a currency, the return of that asset is called **(expected) nominal rate of return**. When instead the value of the asset is based on some broad representative basket of products, the return is called **(expected) real rate of return.**
What really matters is the real rate of return, since the goal of saving is to transfer purchasing power in the future (future consumption), and only the real rate of return can measure the goods and services a saver can buy in the future, as a return to the lower consumption today.

Nominal vs Real

When the asset is a bond, the rate of return is called **interest rate**. The **nominal** interest rate $\mathbf{i_t}$ is the return measured on a given currency, while the **real** interest rate $\mathbf{r_t}$ is the return in terms of a basket of goods.

Let's see what is the specific relation between present vs future (t vs $t+1$) and nominal vs real (i_t vs r_t).

Suppose you decide to save part of your wealth today, in an asset (bond) that has price in terms of €, €P_t today. [1] The nominal interest rate of the bond is given by i_t. The price of the bond tomorrow will be therefore given by equation (1), which describes the relationship between present and future in nominal terms.

$$\underbrace{€P_{t+1}}_{\substack{\text{Nominal}\\ \text{future price}}} = \underbrace{(1+i_t)}_{\text{Nominal return}} \underbrace{€P_t}_{\substack{\text{Nominal}\\ \text{present price}}} \tag{1}$$

Add now the real value to equation 1. As specified above, the real return of an asset gives you the return in terms of a basket of goods. Suppose you want to measure the real value of your bond in terms of home price. The economic problem now is, how much of a house I can buy tomorrow if I invest in a bond with such nominal interest rate i_t. The value of home today and tomorrow must be considered.

$$\underbrace{H_{t+1}}_{\substack{\text{Real value}\\ \text{tomorrow}}} \underbrace{€P_{t+1}}_{\substack{\text{Nominal}\\ \text{future price}}} = \underbrace{(1+i_t)}_{\text{Nominal return}} \underbrace{H_t}_{\substack{\text{Real value}\\ \text{today}}} \underbrace{€P_t}_{\substack{\text{Nominal}\\ \text{present price}}} \tag{2}$$

Note that the real interest rate is the return in terms of basket of goods, in this specific case the basket of goods is the house value. So,

$$H_{t+1} = (1+r_t)H_t \tag{3}$$

Substituting equation 3 in equation 2 we get

$$(1+r_t) = (1+i_t)\frac{P_t}{P_{t+1}} \tag{4}$$

[1] In this problem, as in many other cases in macro, for the sake of simplicity, the bond is a one-period zero-coupon bond. Hence, the face value of the bond is repaid at the time of maturity and the bond does not provide any periodic interest payment (coupons).

The Fisher Equation

Now, recall that the rate of return (or growth rate) of any given asset is the percentage change between its value tomorrow and its value today.

$$g_x = \frac{x_{t+1} - x_t}{x_t}$$

$$g_x = \frac{x_{t+1}}{x_t} - 1$$

$$\frac{x_{t+1}}{x_t} = g_x + 1$$

The same is true for the rate of growth of prices, which is defined as **inflation rate**.

$$\pi_{t+1} = \frac{P_{t+1} - P_t}{P_t} \tag{5}$$

$$\frac{P_{t+1}}{P_t} = \pi_{t+1} + 1 \tag{6}$$

Rearranging equation (6) and substituting into equation (4), it reads

$$(1 + r_t) = (1 + i_t)\left(\frac{1}{\pi_{t+1} + 1}\right)$$

Taking logs to both sides, we obtain the "certainty" version of the Fisher equation.

$$\ln((1 + r_t)) = \ln(1 + i_t) - \ln(\pi_{t+1} + 1)$$

$$i_t \approx r_t + \pi_{t+1} \tag{7}$$

Now, if we introduce uncertainty, we assume that any term in the future is not known in advance, and agent form expectations about this future value. How is and expectation of a variable formed? Through a weighted average. Consider the following basic example. You know that price of a given good today is €1. You don't know the price of that good tomorrow, but you expect it will be either 1.2 with probability 0.4 or 1.5 with probability 0.6. The expected price of the good tomorrow will therefore be:

$$P^e_{t+1} = \mathbb{E}_t(P_{t+1}) = 1.2(0.4) + 1.5(0.6) = 0.9 + 0.48 = €1.38$$

As a consequence, the expected rate of inflation will be

$$\mathbb{E}_t(\pi_{t+1}) = \frac{\mathbb{E}_t(P_{t+1}) - P_t}{P_t} = \frac{1.38 - 1}{1} = 0.38 = 38\%$$

From equation (7) and including expected future variables, the Fisher equation reads as follows, and tells us that the nominal interest rate is approximately given by the real interest rate plus the expected inflation rate.

$$i_t \approx r_t + \mathbb{E}_t(\pi_{t+1}) \tag{8}$$

Put in another way, real interest rate is given by the nominal interest rate minus the expected change in the level of prices.

$$r_t \approx i_t - \pi^e_{t+1} \tag{9}$$