GDP Deflator

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1 GDP - Deflator, nominal and real: relation

Consider the definition of the GDP deflator:

$$\text{GDP}_{\text{DEFL,t}} = \frac{\text{GDP}_{NOM,t}}{\text{GDP}_{REAL,t}} = \frac{€Y_t}{Y_t} \tag{1}$$

As any other measure, a nominal quantity differ by a real one because of its price. Therefore, nominal GDP over real GDP gives a measure of the level of prices. Hence, we can rewrite the above as:

$$P_t = \frac{€Y_t}{Y_t} \tag{2}$$

The ultimate goal of this exercise is to find a relation between the growth rate of nominal and real GDP and inflation.
Let's define the level of inflation as

$$\pi_t = \frac{P_t - P_{t-1}}{P_{t-1}} \tag{3}$$

Take equation (3) and express it for nominal GDP, then divide both sides by nominal GDP at $t-1$:

$$€Y_t = P_t Y_t \tag{4}$$

$$\frac{€Y_t}{€Y_{t-1}} = \frac{P_t Y_t}{€Y_{t-1}} \tag{5}$$

$$\frac{€Y_t}{€Y_{t-1}} = \frac{P_t}{P_{t-1}} \frac{Y_t}{Y_{t-1}} \tag{6}$$

Now consider any generic variable x_t and take its rate of growth in time (between $t-1$ and t).

$$g_x = \frac{x_t - x_{t-1}}{x_{t-1}} = \frac{x_t}{x_{t-1}} - 1$$
$$\frac{x_t}{x_{t-1}} = g_x + 1$$

Take logs to both sides of the last expression. Due to logarithm properties and Taylor approximation:

$$\ln\left(\frac{x_t}{x_{t-1}}\right) = \ln(g_x + 1) \tag{7}$$

$$\ln(g_x + 1) \approx g_x \tag{8}$$

The reason why the approximation holds for small g comes from the Taylor approximation, which in this case in particular is a Maclaurin series. Recall that a Taylor series, for a given function $f(x)$, defined on an open interval $(x_0 - r, x_0 + r)$ and infinitely differentiable on x_0 is given by:

$$f(x) = f(x_0) + \frac{f'(x_0)}{1!}(x-x_0)^1 + \frac{f''(x_0)}{2!}(x-x_0)^2 + \frac{f'''(x_0)}{3!}(x-x_0)^3 + ... = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n$$

The Maclaurin series corresponds to the Taylor series when $x_0 = 0$. Therefore,

$$f(x) = f(0) + \frac{f'(0)}{1!}(x-0)^1 + \frac{f''(0)}{2!}(x-0)^2 + ... = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}(x-0)^n$$

For the logarithm case (natural basis), as of our interest here,

$$\begin{aligned} \ln(1+x) &= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + ... \\ &= \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n} \qquad \text{for } x < 1 \\ \ln(1+x) &\approx x \qquad \text{for small } x \end{aligned}$$

Now apply logarithms to equation (6) and from equation (7), (8), we obtain:

$$\ln\left(\frac{€Y_t}{€Y_{t-1}}\right) = \left(\frac{P_t}{P_{t-1}}\right) + \left(\frac{Y_t}{Y_{t-1}}\right)$$

$$\ln(g_{NOM,t} + 1) = \ln(\pi_t + 1) + \ln(g_{REAL,t} + 1)$$

$$g_{NOM,t} = \pi_t + g_{REAL,t}$$